Operational Characterization of Divisiblity of Dynamical Maps [arXiv:1601.05522]

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In this work, we show the operational characterization to divisibility of dynamical maps in terms of distinguishability of quantum channels. It is proven that distinguishability of any pair of quantum channels does not increase under divisible maps, in which the full hierarchy of divisibility is isomorphic to the structure of entanglement between system and environment. This shows that i) channel distinguishability is the operational quantity signifying (detecting) divisibility (indivisibility) of dynamical maps and ii) the decision problem for divisibility of maps is as hard as the separability problem in entanglement theory. We also provide the information-theoretic characterisation to divisibility of maps with conditional min-entropy.

Dynamics of open quantum systems shares some similarity with entanglement in that the characterisation does have a classical counterpart. In recent years, there has been much progress in understanding, characterising, and detecting divisibility of dynamical maps, the fundamental property that encapsulates and generalises Markovianity of quantum evolution [1, 2, 5, 3], together with careful analysis and classification of non-Markovian quantum evolution (see the collection of papers in [6]). These are of general importance for the study of the interaction between a quantum system and its environment, that is, fundamental phenomena such as dissipation, decay, and decoherence. In the view of quantum information theory, they corresponds to quantum channels conveying information between parties and the properties are connected to information capabilities. In fact, recently it has been shown that complexity of the divisibility problem is computationally intractable, NP-hard [7].

In this work, we present the operational characterisation to divisibility of dynamical maps, more precisely to the refined and general notion k-divisibility, with the unifying idea of quantum channel discrimination. This also merges different approaches of Markovianity in an operational way. Namely, we identify distinguishability of a pair of quantum channels as *information flow* that signifies divisibility of dynamical maps. We show that an infinitesimal increase of distinguishability for an arbitrary pair of channels, hence interpreted as *information backflow*, implies indivisibility, and vice versa. This therefore provides schemes of, both theoretically and practically, detecting indivisible maps including non-Markov processes, similarly to entanglement detection schemes such as entanglement witnesses while the separability problem itself also remains intractable. Our results imply that min-entropy, by which distinguishability is quantified, is hence the information-theoretic tool for the characterisation of divisibility of quantum channels.

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