Estimation in a Deformed Exponential Family

Harsha K V *
kv.harsha@yahoo.com

Subrahamanian Moosath K S
Department of Mathematics
Indian Institute of Space Science and Technology
Thiruvananthapuram-695547, Kerala, India
smoosath@iist.ac.in

In this paper we discuss about certain generalized notions of maximum likelihood estimator and estimation problem in a deformed exponential family. A deformed exponential family has two dually flat structures, U-geometry by Naudts [1] and the χ-geometry by Amari et al. [2]. First we recall the U-estimator defined by Eguchi et al. [3] in a deformed exponential family and its properties. A proof of the generalized Cramer-Rao bound defined by Naudts [1] is given. Then we give a proof of the result that in a deformed exponential family the U-estimator for the dual coordinate in the U-geometry is optimal with respect to the generalized Cramer-Rao bound defined by Naudts.

A generalized MLE called the maximum F-likelihood estimator (F-MLE) is defined in a deformed exponential family. Then we show that F-MLE is given in terms of the dual coordinate in the χ-geometry. Finally we pose an open problem regarding the consistency and efficiency of the F-MLE in a deformed exponential family.

Keywords: Deformed exponential family, U-estimator, F-MLE, U-geometry, χ-geometry

Acknowledgments
We express our sincere gratitude to Prof. Shun-ichi Amari for the fruitful discussions.

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* *Visiting* Department of Mathematics, Indian Institute of Technology Bombay, Powai, Mumbai-400076, Maharashtra, India

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