On Curvature Computations of Multivariate Gaussian Distribution

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The geometry of the multivariate Gaussian distribution has been widely used in literature, [2, 3]. Considering Gaussian distribution as a Riemannian manifold equipped with the Fisher information metric one may compute curvatures of the induced geometry and use it for the purposes of inference in its usual mean and covariance parameters. On the other hand, the Gaussian distribution belongs to the exponential family, and indeed exponential family can be naturally regarded as a Kähler affine manifold with a pair of dually flat affine connections that play an important role in geometric theory of statistical inference. For more information about Kähler affine manifolds and their relation to information geometry apply to [1]. Ones you equip multivariate Gaussian distribution with this structure then affine curvature takes an important role because it carries more information compared to the Riemannian curvature. One can find a detailed discussion about the affine curvature in Shima [4]. It is remarkable that the Riemannian curvature of a Kähler affine metric depends only on the derivatives of the potential function to order at most three, whereas one would expect fourth derivatives of it to appear. Duistermaat gives some explanation for this phenomenon [5]. This property of the Fisher information metric allows us to avoid prolix curvature computations in case of multivariate Gaussian distributions. On contrary to Riemannian curvature, affine curvature does not have this property which make its computation lengthy.

In this work, we develop a simple method to compute different curvatures of multivariate Gaussian distribution and illustrate it for the bivariate case. Then, we

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discuss the importance of affine curvature.

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